DETERMINATION OF SECONDARY CREEP STAGE PARAMETERS OF SHAPED ALUMINA SPINEL REFRACTORY WITH AID OF GENETIC ALGORITHM

Soheil Samadi*, Shengli Jin, Dietmar Gruber, Harald Harmuth
Chair of Ceramics, Montan University of Leoben, Leoben, Austria


# ABSTRACT

Creep is one of the major causes of irreversible strains in refractory linings at high temperatures. The corresponding Norton-Bailey creep parameters of ordinary refractory ceramics can be inversely identified with the Levenberg-Marquardt algorithm. Nevertheless, in most cases, the experimental creep curves are rather fluctuant and diverse. For some cases, the inverse identification could fall into a local minimum if a good initial guess of creep parameters fails. In the current paper, a genetic algorithm is introduced to overcome the local minima of inverse identification. A shaped alumina spinel refractory was used to manifest the benefits and disadvantages of the combined application of the genetic algorithm and the Levenberg-Marquardt algorithm.


# INTRODUCTION

Uniaxial compressive creep [1] and uniaxial tensile creep [2] tests were developed, which allow testing loads up to 20 kN and temperatures up to 1600 °C. The experimental obtained displacement-time curves are used to inversely estimate the Norton-Bailey creep parameters through an optimization algorithm. The local minima could occur during the inverse estimation, caused by the scattering creep results of different specimens and fluctuations in a curve [3].

The Levenberg-Marquardt algorithm (LMA) [4] used for inverse evaluation of refractory creep parameters [1-3] is a gradient based algorithm, which is prone to fall into local minima. As depicted in Fig.1, if the initial point is B, it is more likely to reach a local minimum [5]. Genetic algorithms (GAs) are adaptive heuristic search algorithms, based on the concept of natural selection [6]. In these algorithms, a randomly built initial population is normally used. According to the optimization cost function, the population is checked, and a new population is generated through various stages. The population refinement and the search for the best gene, i.e., final solution, continue until a convergence criterion is met. GAs are used in complex optimization problems. For instance, for unpredictable optimization space [6]. Nevertheless, there are two deficiencies for GAs. Firstly, they can become very time-consuming because the cost function is calculated for each member of the population and several generations. Secondly, the exact global minimum might not be achievable with defined convergence criteria, and often the final solution is in the proximity of the global minimum.

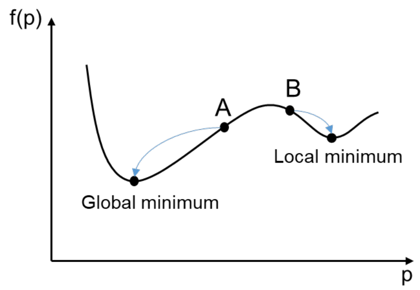


Fig. 1: Dependency of the gradient based optimization approaches on the initial guess

Therefore, in the current study, a GA method was combined with LMA to take its advantages. The goal of this approach (GA+LMA) was to eliminate the effect of initial given parameters on the inversely evaluated creep parameters. The LMA and GA implemented in the commercial software MATLAB were used [7].

The alumina spinel shaped refractory, which is used in the steel ladle barrel zone, was chosen for case study. Uniaxial compressive and tensile creep tests were performed at 1500

°C and 1400 °C respectively. Then the results were used with LMA and GA+LMA approaches to inversely evaluate the creep parameters. The Norton-Bailey creep law was considered for the creep behavior.

## METHODOLOGY

Alumina spinel refractory bricks were chosen for a case study. The batch had 94 wt% alumina, 5 wt% magnesia, and 1 wt% other oxides like silica and iron oxide. The bulk density and open porosity of the bricks were 3.13 g/cm$^3$ and 19 vol%, respectively [3]. For creep testing, the uniaxial compressive creep test described in Ref. [1] and the uniaxial tensile creep test described in Ref. [2] were used. All testing results were reported in Ref. [8].

An entire creep displacement-time curve has three stages, as shown schematically in Fig. 2. The first stage of creep is primary creep, which has a time-dependent strain rate that decreases over time. In the secondary creep stage, the strain rate remains constant. Afterwards, the strain rate gradually increases until the material fails in the tertiary creep stage [9].

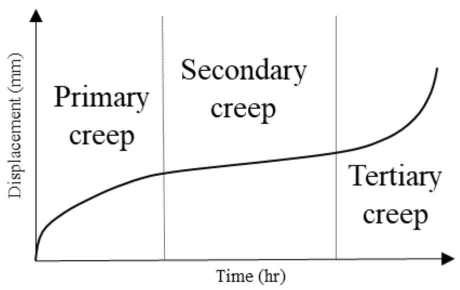


Fig. 2: Schematic of a creep test result [1]

To account for creep behavior, various material models were introduced. The most often used phenomenological creep strain rate model for ordinary refractory ceramics is the Norton-Bailey creep law [10], which can be represented with the following general form:

$$\dot{\varepsilon}_{cr} = K\sigma^n \varepsilon_{cr}^a \qquad \text{(Eq. 1)}$$

where $\varepsilon_{cr}$ is the equivalent creep strain; $K$, n and $a$ are the creep parameters of the material and $\sigma$ denotes the equivalent von Mises stress. $a$ is the strain exponent, which is negative for the primary creep stage, zero for secondary creep stage, and positive for tertiary creep stage. This model was shown to fit the creep testing data of refractory materials well [1-3].

The inverse evaluation approach using LMA is depicted in Fig. 3. After obtaining the creep results from experiments, the three stages of creep are separated using the ruler method, i.e., determination of the secondary creep stage using a ruler. Afterwards, the initial creep parameters are given to LMA, and the algorithm minimizes the cost function, which is the sum of the squared differences of experimental and simulated creep curves.

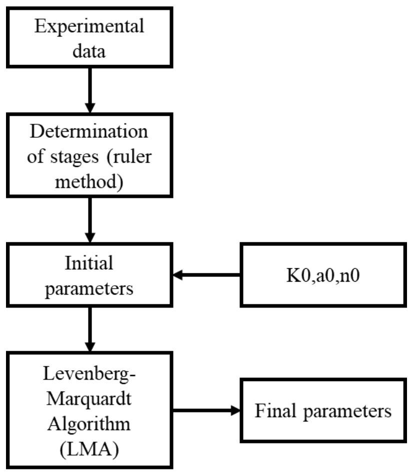


Fig. 3: Flowchart of the inverse evaluation approach using LMA

The flowchart of the new inverse evaluation approach (GA+LMA) is shown in Fig. 4. In this approach, first a random initial population of creep parameters, with a defined population size (50 in the current study), is made. Then the fitness function, i.e., the optimization cost function, is calculated for the population. Afterwards, the convergence criteria are checked. Various convergence criteria can be defined for the GA. In this research, the maximum number of generations was set to 200, and the minimum tolerance of the fitness function was set to $10^{-6}$. If any of these criteria is met, the GA stops, and the best gene is given to LMA. If it is not the case, a new population is generated using GA operators, like selection, crossover, and mutation. Each of these operators can be adjusted in various ways. In the current study, the default parameters of the operators defined in MATLAB were used [7].

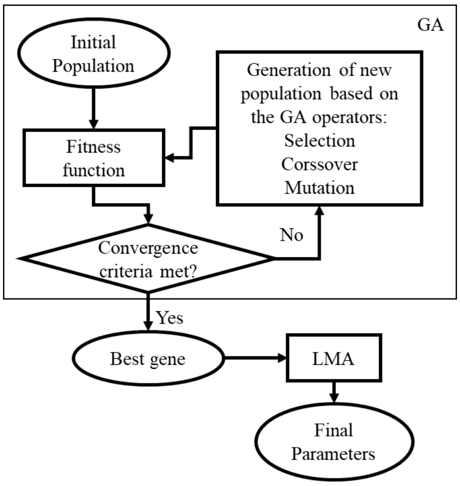


Fig. 4: Flowchart of the inverse evaluation method using GA+LMA

## RESULTS

The compressive creep results at 1500 °C and the tensile creep results at 1400 °C were chosen for this study. These results are shown in Fig. 5 and Fig. 6, respectively. The tests were performed under three constant stresses, and three specimens were tested for each stress, resulting in total 9 tests. An evident scatter was observed in the results of both compressive and tensile creep tests. In addition, in the case of the tensile creep measurements, the fluctuations in the measurement were noticeable compared to the creep strain magnitude.

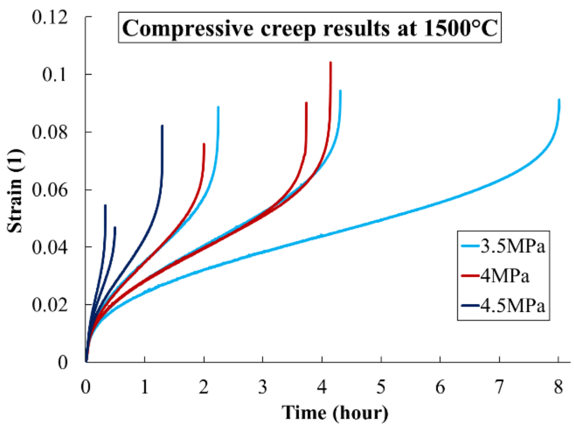


Fig. 5: Uniaxial compressive creep test results of an alumina spinel refractory at 1500°C [8]

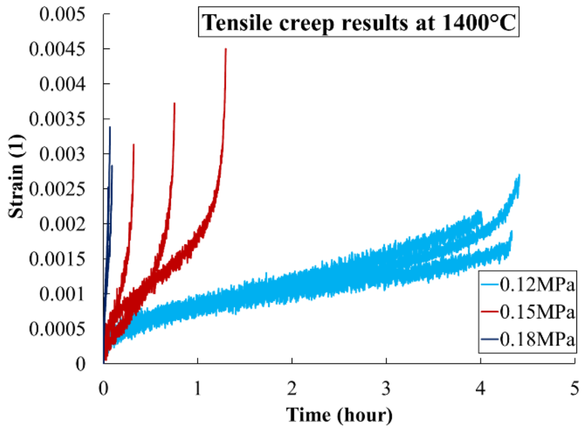


Fig. 6: Uniaxial tensile creep test results of an alumina spinel refractory at 1400°C [8]

Secondary creep stage was the prominent creep stage, especially in the case of tensile creep results. Therefore, the current study evaluated the secondary creep stage parameters. After extraction of this stage from the results, the strain-time data (of all 9 curves) were given to both LMA and GA+LMA approaches. To compare the two approaches, several different initial inputs were given to the LMA approach. In this way, possibility of resulting in local minima was tested.

The secondary compressive creep stage creep parameters of alumina spinel bricks at 1500 °C are shown in Tab. 1. It was observed that different initial parameters resulted in different final parameters using the LMA approach. The GA result was close to the global minimum, and the LMA+GA approach yielded the least minimum shown in the results with the LMA method.

Tab. 1: Comparison of the LMA and LMA+GA inverse evaluation approaches for secondary compressive creep stage parameters

| | **Initial parameters** | | **Final parameters** | | |
|---|---|---|---|---|---|
| | Log(K [$MPa^{-n}$ $s^{-1}$]) | n | Log(K [$MPa^{-n}$ $s^{-1}$]) | n | Fitting residual |
| **LMA** | -2 | 10 | -9.368 | 9.979 | 1.342e+04 |
| | **-8** | **6** | **-8.902** | **5.881** | **6.008e-02** |
| | 0 | 5 | -7.382 | 8 | 1.370e+02 |
| | -2 | 2 | -8.901 | 5.880 | 6.008e-02 |
| **GA result** | | | -8.821 | 5.745 | 6.028e-02 |
| **GA + LMA** | | | **-8.902** | **5.881** | **6.008e-02** |

The secondary stage tensile creep creep parameters of alumina spinel bricks at 1400 °C are shown in Tab. 2. Again, the dependency of the final parameters on the initial parameters were shown. GA+LMA could approach the global minimum, without the need for trial and error in the case of LMA. Nevertheless, the LMA can be used to produce the global minimum when the user knows initial values in the proximity of the final parameters. Finally, the proposed GA+LMA approach had similar calculation time to the LMA approach when a trial and error for the initial given parameter was needed.

Tab. 2: Comparison of the LMA and LMA+GA inverse evaluation approaches for secondary tensile creep stage parameters

| | **Initial parameters** | | **Final parameters** | | |
|---|---|---|---|---|---|
| | Log(K [$MPa^{-n} s^{-1}$]) | n | Log(K [$MPa^{-n} s^{-1}$]) | n | Fitting residual |
| **LMA** | 4 | 2 | 2.097 | -3.290 | 6.094 |
| | 2 | 5 | 2.279 | 10.349 | 1.144e-03 |
| | **0** | **5** | **2.281** | **10.351** | **1.143e-03** |
| | 7 | 2 | -0.383 | 2 | 6.181e+06 |
| | 4 | 4 | -0.848 | 6.528 | 1.938e-03 |
| | -8 | 6 | -8 | 6 | 4.328e-03 |
| **GA result** | | | 4.243 | 12.791 | 1.251e-03 |
| **GA + LMA** | | | **2.281** | **10.351** | **1.143e-03** |

# CONCLUSIONS

The current study suggests an innovative automatized approach for inverse evaluation of creep parameters of refractory materials. The proposed approach is the combination of two optimization algorithm: genetic algorithm and Levenberg-Marquardt algorithm. It was shown that this approach prevents arriving in local minima and supports finding the global minimum. In addition, application of this approach does not require the user knowledge about the creep parameters value range.


# ACKNOWLEDGEMENTS

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